

Development of Symbolic Software to Compute Exact Solutions of Systems of Nonlinear Partial Differential Equations and Lattice Equations

Our research involves the development and implementation of symbolic algorithms for integrability testing and computation of exact solutions of nonlinear partial differential equations (PDEs) and differential-difference equations (DDEs, lattices) [2, 4, 3, 5, 6, 7].

Recently, we designed the packages `PDESspecialSolutions.m` and `DDESspecialSolutions.m` [1] for the computation of travelling wave solutions that can be expressed as polynomials in the tanh and sech functions or the Jacobi elliptic functions (sn and cn). Typical examples of PDEs and lattices that have such solutions are the Korteweg-de Vries and Boussinesq equations and the Toda lattice. The travelling waves (tanh-sech) or cnoidal wave solutions (cn) occur in e.g. fluid flows, plasmas, circuits, optical fibers, bio-genetics.

The students will further develop these Mathematica packages. Specifically, the students will help generalize the method to cover rational (instead of polynomial) solutions. The generalization towards pulse-shaped solutions (sech envelope) with a complex-exponential modulation inside will also be considered. Once implemented, the software would then be able to compute solutions of nonlinear Schrödinger equations which model optical pulses in fibers. Based on the findings for PDEs, the package `DDESspecialSolutions.m` to solve nonlinear DDEs will be further extended and improved.

A key task is solving the nonlinear systems for the coefficients (of the polynomial or rational solutions), which are parameterized by the wave parameters and the parameters in the model equations. We designed a sophisticated solver, yet, it needs further testing and improvements. The students will (i) compare the Ritt-Wu (characteristics sets) method with Groebner basis algorithms, and (ii) test the efficiency of these methods for analyzing and solving systems with parameters.

In summary, the students will help (i) further generalize the hyperbolic and elliptic function methods for solving PDEs and DDEs, (ii) investigate the strengths and weaknesses of algorithms for solving nonlinear algebraic systems with symbolic parameters.

Students will be exposed to modern applied mathematical research, which involves physics (nonlinear waves), pure mathematics (characteristic sets and Groebner basis methods) and computer science (algorithm and software design). As members of a research team, students will learn new mathematical methods and gain experience in advanced programming with Mathematica.

References

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