**Computational Finance and Software Relating to Option Pricing**

Financial derivatives are a major and fast growing area in modern financial markets. For example, according to the *Swaps Monitor*, the size of swaps alone, a particular kind of derivatives, was approximately $9 trillion in 1993, almost the size of the annual gross national income of the United States. The valuation of these derivatives is practically useful, important, and mathematically challenging. Advanced techniques in mathematics have been playing an important role in the understanding and valuation of various derivative securities ever since the first trading of these financial products. With increasing complexity of new and exotic financial products, the demand for new and efficient techniques in mathematics and computation becomes greater and greater.

The major objective of our research is to provide new mathematical tools and efficient computational techniques for an accurate and rational evaluation of modern financial products [1, 2, 3, 10]. To be precise, we shall illustrate our project here for a simple and standard call or put option valuation problem.

A typical call or put option is to give the holder the right to buy (sell) a certain amount of securities at a specified price during some time period. If the option specifies that the holder may exercise the right only at a given expiration date, the claim is called *European*. A more common option, however, is one which may be exercised at any time before the maturity date. These options are termed *American*. In practice, all stock options and futures options are of American type. There are many different ways to specify the payoff function if the option should be exercised by the holder at or before the maturity time. The simplest payoff function for a simple call is merely the difference between the current stock price and the exercise price, provided that the difference is non-negative. Asian options specify the payoff function as the difference between a certain average of the stock price over a specific time interval and the exercise price.

Although the pricing of options has a long history, the theory in mathematical modeling only reached a satisfying level with the celebrated works by Black and Scholes [4] and Merton [8]. The fundamental economic insights that led to the mathematical models are *hedging portfolio*, *replicating portfolio* and *arbitrage-free pricing*. The models, *Black-Scholes equations*, are partial differential equations of parabolic type. A brief introduction to the derivation of the Black-Scholes equation [4] and various extensions can be found from recent books by Duffie [6], Wilmott et al. [13, 14, 15], and Neftci [11]. However, the well-known Black-Scholes formulas and various extensions only handle options of European type. The valuation of American options still presents open challenges to practitioners and researchers in finance.

Our research mainly addresses new mathematics and efficient numerical techniques for the valuation of American options. The new mathematics features a weak variational approach to the time value of the option by using a Hilbert space method. The weak form for the valuation of options opens a door to

the use of finite element methods together with grid local refinement in the approximation of the option pricing function and the free boundary by efficient numerical techniques such as domain decomposition and multigrid methods. In particular, this approach provides a very promising future for the computation of financial products involving multi-assets and securities, as the computational domain will be of multi-dimension in those applications.

A project for the REU program is to develop a web-based software package for various option pricing problems by using the state-of-art numerical methods such as domain decomposition and mixed finite element methods for the Black-Scholes equation. The mixed method will provide a very accurate approximation for the delta hedging formula by approximating the derivative of the pricing function directly. The software package will be written in Java and will be accessible through the internet to the public. Through the development of the proposed package, students will have the opportunity to become acquainted with the topics in option pricing and will eventually be able to work on research projects on option pricing for more complicated financial derivatives.

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