Efficient Spectral Bandwidth Selection in Wavelet Matching

White [3] introduced a method to match synthetic seismograms and seismic traces using spectral methods. Methods for estimating the accuracy of the matching were introduced by Walden and White [2]. The basic idea is the following. We have a seismic trace, $y_t = h_t * x_t + z_t$, and a broadband synthetic reflection sequence, r_t, computed from a well log. The goal is to find a wavelet, h_t, so that $h_t * r_t$ is a reasonable match to the seismic trace. In spectral domain, the data, y_t, are written as

$$Y(\omega) = H(\omega)R(\omega),$$

where Y, H and R are the Fourier transforms of y_t, h_t and r_t, respectively. The estimate of $H(\omega)$ in [3] is obtained by minimizing

$$W_m * \|Y(\omega) - H(\omega)R(\omega)\|^2,$$

where W_m is a spectral window whose bandwidth is determined by a free parameter, m. The bandwidth is proportional to $1/m$. It is assumed that the frequency response of the wavelet is basically constant inside the spectral window. The accuracy of the estimate depends on the type of spectral window and its bandwidth, The smaller the bandwidth, the noisier the estimate but the smaller its bias. Walden and White [2] studied bandwidth selection using the Akaike's information criterion (AIC) and the F-statistic, and concluded that these methods should be used with caution. Still, it is clearly desirable to have an automatic efficient way to choose the bandwidth. However, it is not even clear how to take into account the smoothness of the wavelet to properly choose the bandwidth. We plan to study other, possibly more efficient, bandwidth selection methods based on nonparametric regression methods to define an optimality measure, and/or on time series modeling of data with missing or unequally spaced observations to define cross-validation estimates.

We will also study an alternative to spectral smoothing based on a roughness penalty approach to nonparametric regression. The idea is to expand the wavelet response in some set of basis functions and to include the wavelet smoothness using a roughness penalty controlled by a smoothness parameter. This is in contrast to the approach in [3] where the smoothness is implicitely assumed and fixed. By definition, the wavelet response is a superposition of complex exponentials, but a natural set of functions to expand the frequency response is the set of prolate spheroidal wave functions (PSWF) [1]. In this case, we will define cross-validation methods to choose the smoothing parameter. However, the computational feasibility of the PSWF approach has to be investigated.

References

[1] D.J. Thomson, *Spectrum estimation and harmonic analysis*, Proceedings of the IEEE, 70 (1982), 1055–1096.

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[3] R.E. White, *Partial coherence matching of synthetic seismograms with seismic traces*, Geophysical Prospecting, 28 (1980), 333–358.