Orthogonal Spline Collocation Alternating-Direction Implicit Method for Parabolic and Hyperbolic Problems

In [1, 2], we formulated alternating direction implicit (ADI) schemes for solving linear variable coefficient parabolic and hyperbolic problems in non-divergence form on rectangles with non-homogeneous Dirichlet boundary conditions. Orthogonal spline collocation (OSC) with piecewise Hermite bicubics was used for the spatial discretization and finite differences were used for the temporal approximation. We showed that, for sufficiently small time stepsizes, the schemes are stable and of optimal order accuracy (second order in time with respect to the discrete maximum norm and third order in space with respect to the H^1 norm). We also described efficient implementations of the schemes and presented numerical results demonstrating the accuracy and convergence rates in various norms. In [3], we consider a nonlinear parabolic initial-boundary value problem on a rectangular polygon with variable coefficient Robin's boundary conditions. An approximation to the solution is obtained using an ADI extrapolated Crank-Nicolson scheme in which OSC with piecewise polynomials of arbitrary degree ≥ 3 is used for the spatial discretization and a second order finite difference formula is used for the temporal approximation. For rectangular and L-shaped regions, we describe an efficient B-spline implementation of the scheme and present numerical results demonstrating the accuracy of orders $r+1$, r, and $r-1$ in the spatial norms L^2, H^1, and H^2, respectively.

We propose to extend our work in several directions. First, following [4], we will employ backward differentiation formulas for temporal approximation to obtain OSC ADI schemes of higher order accuracy in both time and space. We also propose to consider ADI Laplace-modified counterparts of the schemes in [1, 2, 3]. Such schemes will require introduction and selection of stability parameters. However, they will allow for a treatment of parabolic problems with very general elliptic parts involving mixed derivatives.

In our numerical experiments, for problems with homogeneous Dirichlet boundary conditions, we observe superconvergence when the initial condition is approximated using the Gauss interpolant rather than the quasi-interpolant suggested by Douglas and Dupont [5] for parabolic equations in a single space variable. We will conduct additional numerical tests to verify whether or not superconvergence occurs for non-homogeneous Dirichlet and, in general, Robin's boundary conditions.

In [3], we propose a very simple approach for approximating time dependent Robin's boundary conditions. We will investigate an application of this approach to finite difference ADI schemes for parabolic problems. A rather involved method for approximating Robin's boundary conditions in such schemes is considered in [6].

Students will be involved in implementing and testing new OSC and finite difference ADI schemes.

References

[1] B. Bialecki and R. I. Fernandes, *An orthogonal spline collocation alternating direction implicit Crank-Nicolson method for linear parabolic problems on rectangles*, SIAM J. Numer. Anal., to appear.

[2] B. Bialecki and R. I. Fernandes, *An orthogonal spline collocation alternating direction implicit method for linear second order hyperbolic problems on rectangles*, submitted.

[3] B. Bialecki and R. I. Fernandes, *Orthogonal spline collocation alternating-direction implicit method for nonlinear parabolic problems on rectangular polygons*, in preparation.

[4] J. H. Bramble, R. E. Ewing and G. Li, *Alternating direction multistep methods for parabolic problems-iterative stabilization*, SIAM J. Numer. Anal., 26 (1989), 904–919.

[5] J. Douglas Jr. and T. Dupont, *Collocation methods for parabolic equations in a single space variable*, Lecture Notes in Math., vol. 385, Springer Verlag, New York, 1974.

[6] B. P. Sommeijer, P. J. Van Der Houwen, and and J. G. Verwer, *On the treatment of time-dependent boundary conditions in splitting methods for parabolic differential equations*, International Journal for Numerical Methods in Engineering, 17 (1981), 335–346.